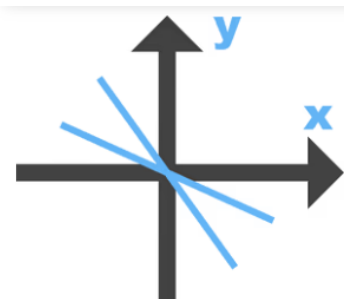

LINEAR MODELING USING THE EQUATION OF A LINE

For the problems in this chapter you may use either of the two forms of a line we've studied: the slope-intercept form,

$$y = mx + b$$

or the point-slope form,

$$y - y_1 = m(x - x_1).$$



I believe you'll find the point-slope form easier because problems in this chapter will provide us with a pair of points on the line, not the y-intercept. In addition, this form is the preferred method in calculus.

EXAMPLE 1: On day 1 the amount, A , of digoxin in the body was 3.2 milligrams (mg). On day 7 the amount was 17 mg. Assume that the amount of digoxin is a linear function of the day, d .

- Find the linear function that expresses the amount, A , as a function of the day, d .
- What does the formula predict for the amount of digoxin on day 10?
- On what day will A be 51.5 mg?
- What does the *slope* of the line represent?
- Solve the formula for d .

Solution: Don't be overwhelmed — almost all of the work is in part **a**. We start by organizing the given data into ordered pairs of the form (d, A) , where d is the day and A is the amount of digoxin:

$$(1, 3.2) \quad (7, 17)$$

Note: d is basically the x , and A is basically the y .

a. The best method we have for finding the equation of the line through the two points is the point-slope method, whose formula is

$$A - A_1 = m(d - d_1)$$

where m is the slope and (d_1, A_1) is any point on the line.

So the only calculation we need is the slope:

$$\begin{aligned} m &= \frac{\Delta A}{\Delta d} && \left[\text{just like } \frac{\Delta y}{\Delta x} \right] \\ &= \frac{17 - 3.2}{7 - 1} \\ &= \frac{13.8}{6} \\ &= \mathbf{2.3} \end{aligned}$$

We can now choose either point; let's choose $(7, 17)$:

$$A - \underline{17} = \mathbf{2.3}(d - \underline{7})$$

This a fine way to write the linear function, but it's actually better to rewrite it in slope-intercept form — this means solve for A and simplify:

$$A - 17 = 2.3(d - 7) \quad (\text{the point-slope form})$$

$$\Rightarrow A - 17 = 2.3d - 16.1 \quad (\text{distribute})$$

$$\Rightarrow A = 2.3d - 16.1 + 17 \quad (\text{add 17 to each side})$$

$$\Rightarrow \boxed{A = 2.3d + 0.9} \quad (\text{the slope-intercept form})$$

b. Just let $d = 10$ in the formula:

$$A = 2.3d + 0.9 = 2.3(10) + 0.9 = 23 + 0.9 = \mathbf{23.09 \text{ mg}}$$

c. Now let $A = 51.5$ in the formula:

$$\mathbf{51.5} = 2.3d + 0.9 \Rightarrow 50.6 = 2.3d \Rightarrow d = 22 \Rightarrow \mathbf{\text{Day 22}}$$

d. The slope, 2.3, represents the average daily increase in the amount of digoxin.

e. $A = 2.3d + 0.9$

$$\Rightarrow A - 0.9 = 2.3d$$

$$\Rightarrow d = \frac{A - 0.9}{2.3}$$

Homework

1. On day 5 the height of a plant is measured to be 38.1 cm. On day 9 the height has increased to 63.7 cm.
 - a. Find the linear formula that expresses the height h of a plant as a function of the day, d .
 - b. Use your formula to predict the height of the tree on day 11.
 - c. On what day is the plant 127.7 cm tall?
 - d. What does the slope of your line represent?
 - e. Solve the formula for d .

2. During a recent snowstorm, there were 12 inches of snow on the ground at 7 p.m., and there were 19 inches at 11 p.m. If you were to graph this data (with time on the horizontal axis and snow on the vertical axis), calculate the slope, and then describe what the slope of the line represents in the context of this problem.
3. A scientist measured the pressure at a depth of 2 ft and determined that it was 15.59 lbs/in^2 (pounds per square inch). At a depth of 10 ft the pressure was 19.15 lbs/in^2 .
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- Find the linear formula that expresses the pressure P as a function of the depth d . [Round to 3 digits.]
 - Use your formula to predict the pressure at a depth of 100 ft.
 - At what depth would we find a pressure of 85.9 lbs/in^2 ?
 - What does the 0.445 in your formula represent?
 - According to the formula, what would be the pressure at sea level ($d = 0$)? Does this make any sense?
 - Solve the formula for d .

Solutions

1. a. $h = 6.4d + 6.1$ b. 76.5 cm c. day 19
- d. It's the amount the plant grows each day (on average)
- e. $d = \frac{h - 6.1}{6.4}$

2. $m = \frac{7}{4} \text{ in/hr}$

The slope represents the average hourly rate of change in the snow level. That is, it's a measure of how fast it's snowing.

3. a. $P = 0.445d + 14.7$

b. 59.2 lbs/in^2

c. 160 ft

d. The increase in pressure for each additional foot below sea level.

e. The pressure would be 14.7 lbs/in^2 . It's not an error. Can you account for the pressure? Hint: It's not water pressure.

f. $d = \frac{P - 14.7}{0.445}$

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**—ALL OF THAT
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MICHELLE OBAMA